Complex Numbers

We do NOT get a real number when we take the square root of a negative number. For example $\sqrt{-9}$ is not a real number because there is no real number that can be squared to a get -9.

i
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Imaginary numbers are used when there is a negative number under a square root. "i" is used to signify an imaginary number. The reason for the name "*imaginary*" numbers is that when these numbers were first proposed several hundred years ago, people could not "*imagine*" such a number.

$i = \sqrt{-1}$ so ... $\sqrt{-4} = \sqrt{-1 \bullet 4} = i\sqrt{4} = 2i$

$i = \sqrt{-1} = i$	$i^5 = i^4 \cdot i$	$i^9 = i$	$i^{13} =$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = -1$	$i^{10} = -1$	$i^{14} =$
$i^3 = -i$	$i^7 = i^4 \cdot i^3 = -i$	$i^{11} = -i$	etc....
$i^4 = 1$	$i^8 = i^2 = 1$	$i^{12} = 1$	

To simplify imaginary numbers with an exponent greater than 3:
1) Divide the exponent by 4
2) The remainder becomes the new exponent
3) Simplify

Examples: i^{13} $i^{12} \cdot i$ $1 \cdot i = i$ i^{12} $(i^4)^3$ $1^4 = 1$ i^{94} $4\overline{)94}$ 23 i^2 -1 i^{27} $4\overline{)27}$ 6 24 3 $i^3 = -i$

To simplify the square root of a negative number:
1) pull out the i
2) simplify the radical

Examples: $\sqrt{-30} = \sqrt{30 \cdot -1}$ $i\sqrt{30}$ $\sqrt{-24}$ $\sqrt{-1 \cdot 24}$ $i\sqrt{4 \cdot 6} =$ $2i\sqrt{6}$ $\sqrt{-45} = \sqrt{i \cdot 45}$ $\sqrt{i \cdot 9 \cdot 5}$ $= 3i\sqrt{5}$

If two square roots with negative numbers are being multiplied: pull out the i BEFORE you multiply!

Examples: $\sqrt{-6} \bullet \sqrt{-10}$
$i\sqrt{6} \cdot i\sqrt{10}$
$i^2\sqrt{60}$
$-1\sqrt{4 \cdot 15}$
$-2\sqrt{15}$

$\sqrt{-8} \bullet \sqrt{2}$
$i\sqrt{8} \cdot \sqrt{2}$
$i\sqrt{16}$
$4i$

Adding/Subtracting: combine like terms

Examples: $(8 - 5i) + (2 + i)$ $10 - 4i$ $(4 + 7i) - (2 - 3i)$ $2 + 10i$

Multiplying with imaginary numbers: NEVER leave i^2 in your answer!

Examples: $(4 + 2i)(3 - 5i)$
$12 - 20i + 6i - 10i^2$
$12 - 14i + 10$
$22 - 14i$

$(4 - i)(3 + 2i)$
$12 + 8i - 3i - 2i^2$
$12 + 5i - 2$
$10 + 5i$

A **complex number** is any number that can be written in the standard form **a + bi**, where a and b are real numbers, and $i = \sqrt{-1}$.
- real numbers are complex numbers with b=0
- pure imaginary numbers are complex numbers with a=0

Complex numbers in equations: Find the values of x and y for which each equation is true.

Examples: $4x - 3yi = 16 + 9i$
$4x = 16$ $x = 4$ $-3yi = 9i$ $y = -3$

$6x + 2yi = -18 + 3i$
$6x = -18$ $x = -3$ $\frac{2yi}{2i} = \frac{3i}{2i}$ $y = \frac{3}{2}$

Every complex number has a **complex conjugate**. The complex conjugate of $a + bi$ is $a - bi$. The conjugate of $3 + 5i$ is $3 - 5i$.

What happens when you multiply conjugates?

Examples: $(2 + i)(2 - i)$
$4 - 2i + 2i - i^2$
$4 - i^2$
$4 - -1 = 5$

$(3 + 5i)(3 - 5i)$
$9 - 25i^2$
$9 - 25(-1)$
$9 + 25 = 34$

Conjugates can be used to rationalize the denominator of a fraction:

Simplify: $\frac{5}{2+3i} \cdot \frac{(2-3i)}{(2-3i)}$
$\frac{10 - 15i}{4 - 9i^2} = \frac{10 - 15i}{13}$
(-1)
$4 + 9$
13

$\frac{3+i}{1-2i} \cdot \frac{(1+2i)}{(1+2i)} = \frac{3 + 6i + i + 2i^2}{1 - 4i^2}$
$\frac{3 + 7i - 2}{1 - 4(-1)} = \frac{3+7i-}{1+4}$
$\frac{3 + 7i - 2}{5}$
$\frac{1 + 7i}{5}$