COMPLETING THE SQUARE

An equation in which one side is a perfect square trinomial, $a^2 + 2ab + b^2 = (a+b)^2$, can be easily solved by taking the square root of each side.

$x^2 - 6x + 9 = 25$

$(x-3)^2 = 25$

$\sqrt{(x-3)^2} = \pm\sqrt{25}$

$(x-3) = \pm 5$

$x = 3 \pm 5$

$x = 8, -2$

What should be added to form a perfect square trinomial?

a. $x^2 + 2x$ $\underline{+1}$ $= (x+1)^2$

b. $x^2 + 16x$ $\underline{+64}$ $= (x+8)^2$

c. $x^2 + 6x$ $\underline{+9}$ $= (x+3)^2$

d. $x^2 - 8x$ $\underline{+16}$ $= (x-4)^2$

e. $x^2 + 10x$ $\underline{+25}$ $= (x+5)^2$

f. $x^2 - 12x$ $\underline{+36}$ $= (x-6)^2$

g. $x^2 - 4x$ $\underline{+4}$ $= (x-2)^2$

h. $x^2 + 8x$ $\underline{+16}$ $= (x+4)^2$

The method of solving by "forcing" a perfect square trinomial is called **completing the square.**

Example 1: Solve $x^2 + 6x + 1 = 0$

$x^2 + 6x + 1 = 0$	Keep all x related terms on one side. Move the constant to the right.
$x^2 + 6x = -1$	
$x^2 + 6x + \square = -1 + \square$	Get ready to create a perfect square on the left. Balance the equation.
$x^2 + 6x + \boxed{9} = -1 + \boxed{9}$	Take half of the x-term coefficient and square it. Add this value to both sides.
$x^2 + 6x + 9 = -1 + 9$	
$x^2 + 6x + 9 = 8$	Factor and write the perfect square on the left.
$(x+3)^2 = 8$	Take the square root of both sides. Be sure to allow for both plus and minus.
$x + 3 = \pm\sqrt{8}$	
$x = -3 \pm \sqrt{8} = -3 \pm 2\sqrt{2}$	Solve for x.
$x = -3 + 2\sqrt{2}; \quad x = -3 - 2\sqrt{2}$	

Completing the square also takes a quadratic in standard form and converts it to vertex form. In the example above: $x^2 + 6x + 1 = y$ is converted to $(x+3)^2 - 8 = y$.

If the coefficient of the x^2 term is not 1, divide each term by that value to create a leading coefficient of one.

Example 2: $2p^2 + 20 = 6p$

$2p^2 + 20 = 6p$	This equation needs some re-arranging.
$2p^2 - 6p = -20$	
$p^2 - 3p = -10$	Divide through by 2 to create the leading coefficient of 1 (for p^2).
$p^2 - 3p + \square = -10 + \square$	Keep all p related terms on one side. Move the constant to the right.
$p^2 - 3p + \boxed{\frac{9}{4}} = -10 + \boxed{\frac{9}{4}}$	Get ready to create a perfect square on the left. Balance the equation.
$p^2 - 3p + \frac{9}{4} = -\frac{31}{4}$	Take half of the p-term coefficient and square it. Add this value to both sides.
$\left(p - \frac{3}{2}\right)^2 = -\frac{31}{4}$	Factor and write the perfect square on the left.
$p - \frac{3}{2} = \pm\sqrt{-\frac{31}{4}}$	Take the square root of both sides. Be sure to allow for both plus and minus.
$p = \frac{3}{2} \pm \sqrt{-\frac{31}{4}} = \frac{3}{2} \pm \frac{\sqrt{-31}}{2}$	Solve for p. Be sure to express the negative radical as an imaginary number.
$p = \frac{3 \pm i\sqrt{31}}{2}$	
$p = \frac{3 + i\sqrt{31}}{2}, \quad p = \frac{3 - i\sqrt{31}}{2}$	

Practice: Solve by completing the square.

1. $x^2 - 8x - 3 = 0$

$x^2 - 8x = 3$

$x^2 - 8x + 16 = 3 + 16$

$x^2 - 8x + 16 = 19$

$(x-4)^2 = 19$

$x - 4 = \pm\sqrt{19}$

$x = 4 \pm \sqrt{19}$

$x = 4 + \sqrt{19} \quad x = 4 - \sqrt{19}$

2. $x^2 + 4x - 7 = 0$

$x^2 + 4x = 7$

$x^2 + 4x + 4 = 7 + 4$

$x^2 + 4x + 4 = 11$

$(x+2)^2 = 11$

$x + 2 = \pm\sqrt{11}$

$x = -2 + \sqrt{11}$

$x = -2 - \sqrt{11}$