Math 3 Notes Solving Polynomial Functions

$(a+b)(a-b)=\underbrace{a^2-b^2}_{\text{difference of squares}}$

$a^3+b^3=(a+b)(a^2-ab+b^2)$
same sign — opposite sign — always

SOAP

$a^3-b^3=(a-b)(a^2+ab+b^2)$
same sign — opposite sign — always

Examples:

A) $8x^3-1=0$

$(2x-1)(4x^2+2x+1)=0$

$2x-1=0$ → $x=\frac{1}{2}$

$4x^2+2x+1=0$

$\frac{-2\pm\sqrt{2^2-4(4)(1)}}{2(4)}$

$\frac{-2\pm\sqrt{4-16}}{8}=\frac{-2\pm\sqrt{-12}}{8}$

$\frac{-2\pm 2i\sqrt{3}}{8}=\frac{-1\pm i\sqrt{3}}{4}$

B) $x^3+27=0$

$(x+3)(x^2-3x+9)$

$x=-3$

$\frac{3\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}$

$\frac{3\pm\sqrt{9-36}}{2}=\frac{3\pm\sqrt{-27}}{2}$

$\frac{3\pm i\sqrt{27}}{2}=\frac{3\pm 3i\sqrt{3}}{2}$

C) $x^4-2x^2=8$

$x^4-2x^2-8=0$

$(x^2-4)(x^2+2)=0$

$(x+2)(x-2)$ $(x^2+2)=0$

$x=2,-2$

$x^2=-2$

$x=\pm i\sqrt{2}$

D) $4x^5-12x^3=40x$

$4x^5-12x^3-40x=0$

$4x(1x^4-3x^2-10)=0$

$4x(x^2-5)(x^2+2)=0$

$4x=0$ → $x=0$

$x^2=5$ → $x=\pm\sqrt{5}$

$x^2=-2$ → $x=\pm i\sqrt{2}$

You Try:

E) $x^3-64=0$

$(x-4)(x^2+4x+16)=0$

$x=4$

$\frac{-4\pm\sqrt{4^2-4(1)(16)}}{2}$

$\frac{-4\pm\sqrt{16-64}}{2}=\frac{-4\pm\sqrt{-48}}{2}=\frac{-4\pm 4i\sqrt{3}}{2}$

$-2\pm 2i\sqrt{3}$

F) $x^4+7x^2+6=0$

$(x^2+6)(x^2+1)=0$

$x^2=-6$ → $x=\pm i\sqrt{6}$

$x^2=-1$ → $x=\pm i\sqrt{1}$ → $x=\pm i$

More examples . . .

G) $x^3+x^2-3x-3=0$

$x^2(x+1)-3(x+1)=0$

$(x^2-3)(x+1)=0$

$x=-1$

$x^2=3$

$x=\pm\sqrt{3}$

H) $x^3-4x^2-2x=-8$

$x^3-4x^2-2x+8=0$

$x^2(x-4)-2(x-4)=0$

$(x^2-2)(x-4)=0$

$x=4$

$x^2=2$

$x=\pm\sqrt{2}$

I) $3x^3+x^2-x+1=0$

$p=1$, $q=3$

$\frac{\pm 1}{\pm 1,\pm 3}=\pm 1,\pm\frac{1}{3}$

$x=-1$

	3	1	−1	1
1		3	4	3
	3	4	3	4

	3	1	−1	1
−1		−3	2	−1
	3	−2	1	0

	3	1	−1	1
1/3		1	2/3	−1/9
	3	2	−1/3	

	3	1	−1	1
−1/3		−1	0	1/3
	3	0	−1	1 1/3

J) $2x^3-x^2=2x-1$

$2x^3-x^2-2x+1=0$

$x^2(2x-1)-1(2x-1)=0$

$(x^2-1)(2x-1)=0$

$x=\frac{1}{2}$

$x^2=1$

$x=\pm\sqrt{1}$

$x=\pm 1$

You Try:

K) $x^3-2x^2-5x+10=0$

$x^2(x-2)-5(x-2)=0$

$(x^2-5)(x-2)=0$

$x^2=5$ → $x=\pm\sqrt{5}$

$x=2$

$3x^2-2x+1$

$\frac{-2\pm\sqrt{(-2)^2-4(3)(1)}}{2(3)}$

$\frac{2\pm\sqrt{4-12}}{6}$ $\frac{2\pm\sqrt{-8}}{6}$

$\frac{2\pm 2i\sqrt{2}}{6}=\frac{1\pm i\sqrt{2}}{3}$