Remainder and Factor Theorems

Remainder Theorem	Factor Theorem
If the polynomial $f(x)$ is divided by $(x-c)$, then the remainder is $f(c)$. (In other words, the remainder is the value of the function at c.)	Let $f(x)$ be a polynomial. If $f(c)=0$, then $(x-c)$ is a factor of $f(x)$. If $(x-c)$ is a factor of $f(x)$, then $f(c)=0$. If $(x-c)$ is a factor of $f(x)$ or if $f(c)=0$, then c is called a zero of $f(x)$.

Example 1: $f(x)=3x^3+4x^2-5x+7$. Find $f(-4)$ using

(a) synthetic division.

$$\begin{array}{r|rrrr} -4 & 3 & 4 & -5 & 7 \\ & & -12 & 32 & -108 \\ \hline & 3 & -8 & 27 & -101 \end{array}$$

(b) the Remainder Theorem.

$3(-4)^3+4(-4)^2-5(-4)+7$

$3(-64)+4(16)+20+7$

$-192+64+27$

-101

Example 2: For each of the problems below, use synthetic division and the Remainder Theorem to find the indicated function value.

A. $f(x)=x^3-7x^2+5x-6$; $f(3)$

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 5 & -6 \\ & & 3 & -12 & -21 \\ \hline & 1 & -4 & -7 & -27 \end{array}$$

$(3)^3-7(3)^2+5(3)-6$

$27-7(9)+15-6$

$27-63+15-6$

-27

B. $f(x)=4x^3+5x^2-6x-4$; $f(-2)$

$$\begin{array}{r|rrrr} -2 & 4 & 5 & -6 & -4 \\ & & -8 & 6 & 0 \\ \hline & 4 & -3 & 0 & -4 \end{array}$$

$4(-2)^3+5(-2)^2-6(-2)-4$

$4(-8)+5(4)+12-4$

$-32+20+12-4$

-4

C. $f(x)=2x^4-5x^3-x^2+3x+2$; $f\left(-\frac{1}{2}\right)$

$$\begin{array}{r|rrrrr} -.5 & 2 & -5 & -1 & 3 & 2 \\ & & -1 & 3 & -1 & -1 \\ \hline & 2 & -6 & 2 & 2 & 1 \end{array}$$

$2(-\frac{1}{2})^4-5(-\frac{1}{2})^3-(-\frac{1}{2})^2+3(-\frac{1}{2})+2$

$.125+.625-.25+-1.5+2$

1

Example 3:

A. Is $x-2$ a factor of $2x^2+3x-14$?

$$\begin{array}{r|rrr} 2 & 2 & 3 & -14 \\ & & 4 & 14 \\ \hline & 2 & 7 & 0 \end{array}$$

yes

B. Is $x+1$ a factor of $5x^3+7x^2-x-3$?

$$\begin{array}{r|rrrr} -1 & 5 & 7 & -1 & -3 \\ & & -5 & -2 & 3 \\ \hline & 5 & 2 & -3 & 0 \end{array}$$

yes

C. Is $m+2$ a factor of m^2-3m-7?

$$\begin{array}{r|rrr} -2 & 1 & -3 & -7 \\ & & -2 & 10 \\ \hline & 1 & -5 & 3 \end{array}$$

No