Rational Expressions: A fraction in which the numerator and/or denominator are polynomials.

Multiplying and Dividing Rational Expressions

Rational Expressions are multiplied, and divided just like fractions.

To multiply fractions:
Factor the numerators & denominators, and then cancel factors.

$$\frac{30}{36}*\frac{21}{60}=\frac{5*6*3*7}{6*6*3*4*5}=\frac{7}{24}$$

To divide fractions:
Multiply by the reciprocal. (*Skip, flip, multiply* -or- *keep, change, flip*)

$$\frac{5}{14}\div\frac{15}{22}=\frac{5}{14}*\frac{22}{15}=\frac{5*2*11}{2*7*3*5}=\frac{11}{21}$$

- A fraction is "**simplified**" when the numerator and denominator have no common factors. To simplify a fraction; factor the numerator and denominator, then cancel the common factors.
- **You can only cancel factors!**

To multiply rational expressions:
Factor the numerators & denominators, and then cancel common factors.

$$\frac{(m-3)^2}{m^2-6m+9}\cdot\frac{m^3-9m}{m^2-9}=\frac{(m-3)^2m(m-3)(m+3)}{(m-3)^2(m-3)(m+3)}=m$$

To divide rational expressions:
Multiply by the reciprocal. (*Skip, flip, multiply* -or- *keep, change, flip*)

$$\frac{x^2+7x+10}{x-6}\div\frac{x+5}{x^2-36}=\frac{x^2+7x+10}{x-6}\bullet\frac{x^2-36}{x+5}=\frac{(x+2)(x+5)(x+6)(x-6)}{(x-6)(x+5)}=(x+2)(x+6)$$

In the rational expression $\frac{x(x-3)}{x+2}$ you CANNOT cancel the x in the numerator (which is a factor) with the x in the denominator because the x in the denominator is NOT A FACTOR (it doesn't multiply everything else in the denominator)! You can only cancel common terms when BOTH are factors!

- **Factors multiply everything else in the numerator/denominator!**

Notes : Multiply & Divide Rational Expressions

The strategies for multiplying and dividing rational expressions are similar to the strategies you learned about multiplying and dividing fractions. The main difference is that when you have a polynomial expression, you must first factor.

Multiplying Rational Expressions:

1. Write all polynomials in factored form.
2. Check to see if any factors can be canceled before multiplying. *("Cancel factors low and high")*
3. Multiply remaining factors straight across.
4. Answers may be left in factored form.

Dividing Rational Expressions:

1. Write all polynomials in factored form.
2. *"Same-Change-Flip":* Leave the first fraction the same, change to multiply, flip the second fraction.
3. Cancel factors if possible. (This can ONLY be done AFTER you flip the second fraction.)
4. Multiply remaining factors straight across.
5. Answers may be left in factored form.

Examples:

A. $\frac{2a^4+8a^2}{a-2}\cdot\frac{b-3}{8a^3b^2+32ab^2}$

$\frac{2a^2(a^2+4)}{a-2}\cdot\frac{b-3}{8ab^2(a^2+4)}$

$\frac{a(b-3)}{4b^2(a-2)}$

B. $\frac{7a}{9b}\cdot\frac{63b^3}{35a^2}=\frac{7b^2}{5a}$

C. $\frac{y+2}{y^2+5y}\div\frac{y^2-2y}{y^2-25}$

$\frac{y+2}{y(y+5)}\cdot\frac{(y+5)(y-5)}{y(y-2)}=\frac{y+2}{y^2}\cdot\frac{y-5}{y-2}$

D. $\left(\frac{2xy}{w^2}\right)^3\div\frac{24x^2}{w^5}$

$\frac{8x^3y^3}{w^6}\cdot\frac{w^5}{24x^2}$

$\frac{xy^3}{3w}$

E. $\frac{\frac{8x^2-72}{5x+10}}{\frac{4x-12}{5x}}$

$\frac{8x^2-72}{5x+10}\div\frac{4x-12}{5x}$

$\frac{8x^2-72}{5x+10}\cdot\frac{5x}{4x-12}$

$\frac{8(x+3)(x-3)}{5x+10}\cdot\frac{5x}{4(x-3)}$

$\frac{2x(x+3)}{x+2}$