

# Solving Radical Equations

A RADICAL EQUATION is an equation that has a variable in a radical or has a variable with a fractional exponent.

Examples: Which of the following are radical equations?

A)  $3 + \sqrt{x} = 10$

B)  $(x-2)^2 = 25$

C)  $\sqrt{3} + x = 10$

## Solving Radical Equations With Variables in the Radicand

Steps:

1. Isolate the radicand.
2. Raise both sides of the equation to the same power. The power should be the same value as the index of the radical.
3. Solve for x.

Examples: Solve each equation.

D)  $\sqrt{5x+1} - 6 = 0$

$\sqrt{5x+1} = 6$

$5x+1 = 36$

$5x = 35$

$x = 7$

E)  $3\sqrt{x+3} = 15$

$\frac{3\sqrt{x+3}}{3} = \frac{15}{3}$

$\sqrt{x+3} = 5$

$x+3 = 25$

$x = 22$

F)  $\sqrt[3]{2x+3} - 5 = 0$

$\sqrt[3]{2x+3} = 5$

$2x+3 = 125$

$2x = 122$

$x = 61$

## Solving Radical Equations with Rational Exponents

Steps:

1. Isolate the parenthesis with the rational exponent.
2. Raise both sides to the reciprocal power. If the denominator is even, put absolute value bars around the result.
3. Simplify and solve for x.

Examples: Solve each equation.

G)  $2(x-2)^{\frac{1}{2}} = 50$

$(x-2)^{\frac{1}{2}} = 25$

$x-2 = 125$

$x = 127$

$x = 127, -123$

H)  $(x+5)^{\frac{3}{2}} = 4$

$x+5 = \pm 8$

$x = 3, -13$

$x = 3, -13$

I)  $3(x-2)^{\frac{2}{3}} = 24$

$(x-2)^{\frac{2}{3}} = 8$

$x-2 = 12$

$x = 18$

Checking for Extraneous Solutions  
Extraneous Solutions (answers that do not work when plugged back into the original equation) can occur when you have more than one variable.

Examples: Solve each equation.

J)  $\sqrt{x-3} + 5 = x$

$(\sqrt{x-3})^2 = (x-5)^2$

$x-3 = x^2 - 10x + 25$

$0 = x^2 - 11x + 28$

$(x-7)(x-4)$

Solving Equations with Multiple Radicals  
If an equation contains two radical expressions (or two terms with rational exponents), isolate one of the radicals (or one of the terms).

K)  $\sqrt{11x+3} - 2x = 0$

$\sqrt{11x+3} = 2x$

$11x+3 = 4x^2$

$0 = 4x^2 - 11x - 3$

$(4x+1)(x-3)$

$x = 3$

L)  $(x+3)^{\frac{1}{2}} - 1 = x$

$(x+3)^{\frac{1}{2}} = 1+x$

$x+3 = x^2 + 2x + 1$

$0 = x^2 + 1 - 2x - 3$

$(x+2)(x-1)$

$x = 1$

Examples: Solve each equation.

M)  $\sqrt{x+2} = \sqrt{3x-10}$

$x+2 = 3x-10$

$12 = 2x$

$x = 6$

N)  $\sqrt{x-8} + \sqrt{x} = 4$

$\sqrt{x-8} = 4 - \sqrt{x}$

$x-8 = 16 - 8\sqrt{x} + x$

$-8 = -8\sqrt{x}$

$1 = \sqrt{x}$

$x = 1$

$(4-\sqrt{x})(4+\sqrt{x})$

$16 - 8\sqrt{x} + x$

$x = 9$

P)  $(2x-1)^{\frac{1}{3}} = (x+1)^{\frac{2}{3}}$

$(2x-1)^{\frac{1}{3}} = x+1$

$(2x-1)^2 = (x+1)^3$

$4x^2 - 4x + 1 = x^3 + 3x^2 + 3x + 1$

$4x^2 - 4x = x^3 + 3x^2 + 3x$

$4x^2 - 5x = 0$

$x(4x-5) = 0$

$x = 5/4$

O)  $\sqrt{x-9} - \sqrt{x-6} = 3$

$(\sqrt{x-9})^2 = (3 + \sqrt{x-6})^2$

$x-9 = 9 + 6\sqrt{x-6} + x-6$

$x-9 = 6\sqrt{x-6} + x+3$

$x-12 = 6\sqrt{x-6} + x$

$-12 = 6\sqrt{x-6}$

$2 = \sqrt{x-6}$

$4 = x-6$

$x = 10$