

**Notes: Variations**

**Variations**

A variation is a relationship between variables in which there is always some constant of variation (k). The three types of variation are direct, inverse and joint. The table below shows how to write an equation involving variation.

| Variation | Equation Form     |
|-----------|-------------------|
| Direct    | $y = kx$          |
| Inverse   | $y = \frac{k}{x}$ |
| Joint     | $y = kxz$         |

**Examples:**

A) Suppose that y varies directly with x. If  $y = 14$  when  $x = 5$ , then find x when  $y = 28$ .

$$y = kx$$

$$14 = k \cdot 5$$

$$\frac{14}{5} = k$$

$$28 = \frac{14}{5} x$$

$$28 \cdot \frac{5}{14} = x$$

$$x = 10$$

B) Suppose that y varies inversely with the square of x. Find y when  $x = 12$  if  $y = -6$  when  $x = 2$

$$y = \frac{k}{x^2}$$

$$-6 = \frac{k}{2^2}$$

$$-6 \cdot 4 = k$$

$$-24 = k$$

$$y = \frac{-24}{x^2}$$

$$y = \frac{-24}{12^2}$$

$$y = \frac{-1}{6}$$

C) Suppose that y varies jointly with x and z and inversely with p when  $x = 9, z = 2, p = 6$  and  $y = 15$ . Find x when  $y = 2, z = 1$  and  $p = 10$ .

$$y = \frac{kxz}{p}$$

$$15 = \frac{k(9)(2)}{6}$$

$$90 = 18k$$

$$k = 5$$

**You Try:**

D) Suppose that z varies jointly with x and y and when  $x = 3$  then  $y = 12$  and  $z = 2$ . Find z when  $x = 6$  and  $y = 4$ .

$$z = (5) \frac{x(1)}{10}$$

$$20 = 5x$$

$$x = 4$$

$$z = x y k$$

$$2 = 3(12)k$$

$$2 = 36k$$

$$k = \frac{2}{36} = \frac{1}{18}$$

$$z = 6(4)(\frac{1}{18})$$

$$z = \frac{24}{18} = \frac{4}{3}$$

E) Suppose that z varies directly with x and inversely with the cube of y. When  $x = 8$  and  $y = 2, z = 3$ . Find z when  $x = 6$  and  $y = 4$ .

$$z = \frac{kx}{y^3}$$

$$3 = \frac{8k}{2^3}$$

$$24 = 8k$$

$$3 = k$$

$$z = \frac{6(3)}{4^3}$$

$$z = \frac{18}{64}$$

$$z = \frac{9}{32}$$

F) The distance a spring stretches varies directly with the amount of weight that is hanging on it. A weight of 2.5 pounds stretches a spring 18 inches. Find the stretch of a spring when a weight of 6.4 pounds is hanging on it.

$$y = kx$$

$$18 = k(2.5)$$

$$7.2 = k$$

$$y = (7.2)(6.4)$$

$$y = 46.08$$

G) When a person swims underwater, the pressure in his or her ears varies directly with the depth at which he or she is swimming. At 10 feet, the pressure is about 4.3 pounds per square inch (psi).

a) Find an equation of direct variation that represents this situation.

$$P = k d$$

$$4.3 = k(10)$$

$$k = .43$$

b) Find the pressure if the depth is 60 feet.

$$P = .43(60) = 25.8 \text{ psi}$$

c) It is unsafe for amateur divers to swim where the water pressure is more than 65 psi. How deep can an amateur diver safely swim?

$$65 = .43 d$$

$$151.2 \text{ ft} = d$$

H) When in kick boxing, it is found that the force, f, needed to break a board, varies inversely with the length, l, of the board. If it takes 5 lbs of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long?

$$f = \frac{k}{l}$$

$$5 = \frac{k}{2}$$

$$10 = k$$

$$f = \frac{10}{6}$$

$$f = 1.67$$

$$f = 1.7$$