

Rational Unit Review

olve each variation word problem.

1. Suppose that y varies directly with x and inversely with z when $y = 40$, $x = 8$ and $z = 2$. Write the equation that models the relationship. Then find y when $x = 4$ and $z = 2$. ($y = \frac{10x}{z}$, 20)

$y = \frac{xk}{z}$
 $40 = \frac{8k}{2}$
 $80 = 4k$
 $k = 20$
 $y = \frac{4(20)}{2} = \frac{80}{2} = 40$

2. Radiation machines, used to treat tumors, produce an intensity of radiation that varies inversely as the square of the distance from the machine. At 3 meters, the radiation intensity is 62.5 milliroentgens per hour. What is the intensity at a distance of 2.5 meters? (90 milliroentgens)

$R = \frac{k}{d^2}$
 $62.5 = \frac{k}{3^2}$
 $k = 562.5$
 $R = \frac{562.5}{2.5^2} = \frac{562.5}{6.25} = 90$

3. The number of hours h that it takes m men to assemble x machines varies directly as the number of machines and inversely as the number of men. If four men can assemble 12 machines in four hours, how many men are needed to assemble 36 machines in eight hours? (6 men)

$h = \frac{kx}{m}$
 $4 = \frac{k(12)}{4}$
 $16 = k(12)$
 $k = \frac{4}{3}$
 $8 = \frac{(\frac{4}{3})x}{m}$
 $8m = \frac{4x}{3}$
 $24m = 4x$
 $6m = x$
 $m = 6$

4. The kinetic energy (E) of a moving object varies jointly with its mass (m) and the square of its velocity (v). If an object weighing 90 kilograms and moving with a velocity of 20 meters per second has a kinetic energy of 4500 joules, find its kinetic energy when the mass stays the same and the velocity is 32 meters per second. (11520 joules)

$E = mv^2k$
 $4500 = 90(20)^2k$
 $125 = k$
 $E = mv^2k$
 $E = 90(32)^2(125)$
 $E = 11,520$

find the points of discontinuity (vertical asymptotes and holes).

5. $y = \frac{(x+3)(x-4)}{x(x-5)(x+1)}$
 holes: $x=0$
 $x=5$
 $x=-1$
 Find the horizontal asymptotes.
 $y = \frac{3x^2-2x+1}{4x^2-2}$
 none
6. $y = \frac{1}{(x+2)(x-3)}$
 VA: $x = -2, x = 3$
 hole: none
7. $y = \frac{x(x+6)}{(x-8)}$
 VA: $x = 8$
 hole: $x = 0$
9. $y = \frac{-x^2-2}{-7x^2}$
 $y = 0$
10. $y = \frac{3x+5}{2x^2-3}$
 $y = 0$

Simplify each rational expression and state any restrictions. (REDUCE!)

11. $\frac{x^2-2x-3}{x^2-9x+8}$
 $\frac{(x-3)(x+1)}{(x-8)(x-1)}$
 $x \neq 8, x \neq 1$

12. $\frac{x^2}{x+5} \cdot \frac{x^2+7x+10}{x}$
 $\frac{x^2}{x+5} \cdot \frac{(x+5)(x+2)}{x}$
 $\frac{x^2(x+2)}{x(x+5)}$
 $\frac{x(x+2)}{x+5}$
 $x \neq 0, x \neq -5$

13. $\frac{x-3}{x^2+7x+12} \div \frac{3-x}{x+4}$
 $\frac{x-3}{(x+3)(x+4)} \cdot \frac{x+4}{3-x}$
 $\frac{x-3}{(x+3)(x+4)} \cdot \frac{x+4}{-(x-3)}$
 $\frac{-1}{x+3}$
 $x \neq -3, x \neq -4, x \neq 3$

14. $\frac{x^4-2x^2-3}{x^4-5x^2-6} \div \frac{x^2-3}{x+1}$

$\frac{(x^2-3)(x^2+1)}{(x-6)(x^2+1)} \cdot \frac{x+1}{x^2-3}$
 $\frac{x+1}{x-6}$
 $x \neq 6$

17. $\frac{x^2+5x-14}{3x-6} \cdot \frac{x-1}{x^2-6}$

$\frac{(x+7)(x-2)}{3(x-2)} \cdot \frac{x-1}{x(x-2)}$
 $\frac{x+7}{3} \cdot \frac{x-1}{x(x-2)}$
 $\frac{(x+7)(x-1)}{3x(x-2)}$

15. $\frac{x^2-1}{10x} \cdot \frac{3x^2-x-2}{3x^2-x-2}$

$\frac{(x-1)(x+1)}{10x} \cdot \frac{(3x+2)(x-1)}{(3x+2)(x-1)}$
 $\frac{(x-1)(x+1)}{10x}$
 $x \neq 0$

18. $\frac{5}{4} \div \frac{7}{6}$

$\frac{20}{24ab} = \frac{21b}{24ab}$
 $20 = 21b$

16. $\frac{x-5}{x+1} \cdot \frac{3}{3+x}$

$\frac{2}{x-5} \cdot \frac{1}{3+x}$
 $\frac{2}{(x-5)(3+x)}$
 $\frac{2x}{x^2-2x-15}$

19. $\frac{x}{3x-2} \cdot \frac{x}{(x+1)(x-1)}$

$\frac{x(x+1)-3x}{3(x-1)(x+1)}$
 $\frac{x^2+x-3x}{3(x-1)(x+1)}$
 $\frac{x^2-2x}{3(x-1)(x+1)}$

21. $\frac{1}{2} \div \frac{3}{20}$

$\frac{2}{60} \cdot \frac{20}{10}$
 $\frac{2}{60} \cdot \frac{20}{10} = \frac{4}{60} = \frac{1}{15}$

22. $\frac{x^2+4x-21}{x+3} \div \frac{x+3}{x^2-4x+3}$

$\frac{(x+7)(x-3)}{x+3} \cdot \frac{(x-3)(x-1)}{(x-3)(x-1)}$
 $\frac{(x+7)(x-3)}{x+3}$
 $x \neq -3$

Solve the following Rational Equations. Find common denominators for each term, then compare numerators to solve. Don't forget you can take a shortcut by cross-multiplying if there is only one fraction on either side of the equation.

23. $\frac{5}{x-1} = \frac{2}{x+1}$
 $5(x+1) = 2(x-1)$
 $5x+5 = 2x-2$
 $3x = -7$
 $x = -\frac{7}{3}$
 $x \neq 0$

24. $\frac{x-15}{x+5} = \frac{x-12}{x}$
 $x(x-15) = (x+5)(x-12)$
 $x^2-15x = x^2-7x-60$
 $0 = 8x-60$
 $0 = 4(2x-15)$
 $2x = 15$
 $x = \frac{15}{2}$
 $x \neq 0, x \neq -5$

25. $\frac{3-x}{x^2-2x-6} = \frac{2}{x^2-2}$

$\frac{3-x}{(x-3)(x+2)} = \frac{2}{(x-3)(x+2)}$
 $3-x-2(x-3) = 2(x-3)$
 $3-x-2x+6 = 2x-6$
 $-3x+9 = 2x-6$
 $9 = 5x-6$
 $15 = 5x$
 $x = 3$
 $x \neq 3, x \neq -2$

26. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$
 $\frac{12}{x} = \frac{3}{2} - \frac{3}{4}$
 $\frac{12}{x} = \frac{3}{4}$
 $48 = 3x$
 $16 = x$

27. $\frac{5}{x+2} - \frac{2}{x-4} = \frac{2}{x-2}$
 $5(x-2)(x-4) - 2(x+2)(x-4) = 2(x+2)(x-4)$
 $5(x^2-6x+8) - 2(x^2-2x-8) = 2(x^2-2x-8)$
 $5x^2-30x+40 - 2x^2+4x+16 = 2x^2-4x-16$
 $3x^2-26x+56 = 2x^2-4x-16$
 $x^2-22x+72 = 0$
 $(x-4)(x-18) = 0$
 $x = 4, x = 18$

- Answers: 5) V.A.: $x = 0$, $x = -1$; holes: $x = 5$ 6) V.A.: $x = -2$, $x = 3$; holes: none 7) V.A.: $x = 8$; holes: $x = 0$ 8) none 9) $y = \frac{1}{3}$
- 10) $y = 0$ 11) $\frac{x(x+3)}{x^2-9x+8}$; $x \neq \pm 2\sqrt{2}$, $w \neq \pm 1$ 12) $x(x+2)$; $x \neq -5$, $x = 0$ 13) $\frac{x-1}{x+1}$; $x \neq -4$, $x = 3$ 14) $\frac{x^2-2x-3}{x^2-2x-6}$; $x \neq 6$ 15) $\frac{3x^2-x-2}{3x^2-x-2}$; $x \neq 0$ 16) $\frac{x^2-2x-15}{x^2-2x-15}$; $x \neq 0$ 17) $x+7$ 18) $\frac{20-21b}{24ab}$ 19) $\frac{x^2-2x}{3(x-1)(x+1)}$ 20) $\frac{3(x-2)}{x-4}$ 21) -1 22) $x+7$ 23) $\frac{1}{15}$ 24) $\frac{15}{2}$ 25) $\frac{3-x}{x+2}$ 26) 16 27) $\frac{1}{3}$