Interest Word Problems

Simple Interest vs. Compound Interest

Simple Interest: interest is only paid on the original amount

$$I = Prt$$

I = interest, P = principal, r = rate, t = time

Compound Interest: "interest on interest": Interest is added to original principal amount, and this is the new value of the principal for the next time period

$$A = P(1+\frac{r}{n})^{nt}$$

A = final amount, P = principal, r = rate, t = time, n = number of times compounded

Compounded Continually: the balance grows by a small amount every instant

$$A = Pe^{rt}$$

A = final amount, P = principal, r = rate, t = time

Example 1: If $500 is invested at 6 % interest, how much capital is accumulated after 5 years with:

a) Simple Interest $I = Prt$ $\quad I = 500(.06)(5)$ $\quad I = \$150$

b) Annual Compounding $A = P(1+\frac{r}{n})^{nt}$ $\quad A = 500(1+\frac{.06}{1})^{1(5)} = (500)(1.06)^5$ $\quad A = 669.11$

c) Quarterly compounding

$A = P(1+\frac{.06}{4})^{4(5)} = 500(1+\frac{.06}{4})^{20} = 500(1.015)^{20} = \$673.43$

d) Daily Compounding

$A = P(1+\frac{.06}{365})^{365 \cdot 5} = 500(1.000168539)^{1825} = \$680.05 = 674.91$

e) Continuous Compounding

$A = Pe^{rt} = 500e^{.06 \cdot 5} = 500e^{.3} = \$674.93$

Example 2: What interest rate would a $500 investment have to earn in order to double in four years? (Assume continuous compounding)

$A = Pe^{rt}$

$\frac{1000 = 500e^{r(4)}}{500}$

$2 = e^{4r}$

$\ln 2 = 4r$

$r = \frac{1}{4}\ln 2$

$r = .1733$

$r = 17.3\%$

Example 3: How long would it take for a $395 investment that is compounded continuously at a rate of 15% to reach a total amount of $1500?

$A = Pe^{rt}$

$\frac{1500 = 395e^{(.15t)}}{395}$

$\ln(\frac{1500}{395}) = .15t$

$\ln(\frac{1500}{395})$ $= t$

Growth and Decay Word Problems

Growth and Decay

Use the following model for appreciation, depreciation and population growth.

$$y = ab^x$$

y = final amount, a = principal, b = 1 ± rate, x = time

- For growth problems, the rate (in decimal form) is added to b.
- For decay problems, the rate (in decimal form) is subtracted from b.

Examples:

A. The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%. Find the population in 2010.

$y = 6.08(1+.0126)^{10}$

$y = 6.89$ billions

B. Jim bought a new car for $18,000. After 4 years, the car was valued at $7500. What was the rate of depreciation?

$7500 = 18000(1-r)^4$

$\frac{7500}{18000} = (1-r)^4$

$(\frac{75}{180})^{\frac{1}{4}} = 1-r$

$r = 1+(\frac{75}{180})^{1/4}$

$r = .1.8\%$

Half-Life

$$y = ab^x$$

y = final amount, a = initial amount, b = (1/2), x = (time/half-life)

Examples:

C. Radium has a half-life of 1620 years. A 20 gram sample is sealed in a box. How many grams will be left in 5000 years?

$y = ab^x$

$y = 20(.5)^{\frac{5000}{1620}}$

$y = 2.35g$

D. The half life of element X is 57 minutes. Starting with 35 milligrams, how long will it take to decay to 5 mg?

$y = ab^x$

$5 = 35(.5)^{\frac{x}{57}}$

$\frac{5}{35} = .5^{\frac{x}{57}}$

$\log\frac{1}{7} = \frac{x}{57}\log .5$

$\frac{\log\frac{1}{7}}{\log .5} = \frac{x}{57} = \log_{.5} x = \log\frac{1}{7}(57)$

$x = 57\log_{.5}\frac{1}{7}$

$x = 57\log_{.5}\frac{1}{7}$