Geometric Sequences & Series

Geometric Sequence: $a_n = a_1(r)^{n-1}$, where a_n is the nth term, r is the common ratio, & a_1 is the 1st term.

Example 1: Find the fifth term of the sequence given $a_1 = 8$ and $r = 3$.

$a_n = a_1(r)^{n-1}$ $a_5 = 8(3)^{5-1}$

$a_5 = 648$

You Try!

Find the seventh term of the sequence defined by $a_1 = 5$ and $r = 2$.

$a_n = a_1(r)^{n-1}$ $= 5(2)^6$

$a_7 = (5)(2)^{7-1}$ $a_7 = 320$

Example 2: Write both the recursive and explicit representations of the pattern and state if it arithmetic, geometric, or neither. Then find the 15th term.

a) $\frac{1}{243}, \frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots$ Geometric (·3)

R: $a_n = 3a_{n-1}$

E: $a_n = \frac{1}{243} \cdot 3^{n-1}$

$a_{15} = \frac{1}{243} \cdot 3^{15-1} = 19{,}683$

b) 53, 47, 41, 35, ... Arithmetic (-6)

R: $a_n = a_{n-1} - 6$

E: $a_n = 53 - 6(n-1)$

$a_{15} = 53 - 6(15-1)$

$a_{15} = -31$

c) 2, 3, 5, 9, 17, 33, 65, ... Neither x2-1

R: $a_n = 2a_{n-1} - 1$

E: $a_n = 1 + 2^{n-1}$

$a_{15} = 1 + 2^{15-1}$

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Consider the sequence 2, 4, 8, 16, 32, ... The sum of the first five terms, denoted S_5, is

2 + 4 + 8 + 16 + 32 = 62 S_5

Sum of the First n Terms of a FINITE Geometric Series

$$S_n = a_1 \frac{1-r^n}{1-r}$$

Example 1:

Given the series 3 + 4.5 + 6.75 + 10.125 + ..., find S_{10} to the nearest tenth. x1.5

n=10 $S_n = a_1 \frac{1-r^n}{1-r}$ $S_{10} = 339.99$

$a_1 = 3$

r=1.5 $S_{10} = 3\frac{(1-1.5^{10})}{1-1.5}$

You Try!

Given the series 400 + 300 + 225 + 168.75 + ... find S_{10} to the nearest tenth. x.75

n=10 $S_n = a_1 \frac{1-r^n}{1-r}$ $S_{10} = 1509.9$

$a_1 = 400$

r=.75 $S_{10} = 400 \frac{1-.75^{10}}{1-.75}$

recursive = need previous # (now-next)

explicit = can be used for any term

just need initial amount.

Summation Notation (or Sigma Notation): This means "Add" → $\sum$ It's called "Sigma"

Summation Notation

$$a = \sum_{n=1}^{10} a_n = a_1 + a_2 + \ldots + a_9 + a_{10}$$

Example 2: Find the sum of the given geometric series.

a) $\sum_{n=1}^{11} 4\left(-\frac{1}{3}\right)^{n-1}$ $a_1 = 4$ $r = -\frac{1}{3}$ $n = 11$

$S_{11} = 4\frac{1-(-\frac{1}{3})^{11}}{1-\frac{-1}{3}}$ $S_{11} = 71.03$

b) 5 + 15 + 45 + ... + 98415

$a_1 = 5$ $a_n = a_1 \cdot r^{n-1}$

$r = 3$ $98415 = 5 \cdot 3^{n-1}$

$n = ?$ $19683 = 3^{n-1}$

$\log_3 19683 = n-1$

$\log_3 19683 + 1 = n$

You Try!

$\sum_{n=1}^{6} 2(3)^{n-1}$ n=6 r=3 $a_1 = 2$ $S_6 = 2\frac{1-3^6}{1-3}$ $S_6 = 728$

Sum of an INFINITE Geometric Series

$$\sum_{n=1}^{\infty} a_1 \cdot r^{n-1} = \frac{a_1}{1-r}, \text{ if and only if } |r| < 1$$

If an infinite series has a sum, it converges.

If it does not, it diverges.

Example 3: Find 5 + 20 + 80 + 320 + 1280 + ...

We need first to find a and r.

a=5

r=4

Diverge or Converge? How do we know? r bigger than 1

Example 4: Determine if the following series converge. If they do converge, find the sum.

a) $\sum_{n=1}^{\infty} 3(0.75)^{n-1}$

Converge

$\frac{a_1}{1-r} = \frac{3}{1-.75}$

$\frac{3}{.25}$

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b) $\sum_{n=1}^{\infty}\left(-\frac{4}{5}\right)^{n-1}$

Converge

$\frac{a_1}{1-r} = \frac{1}{1+\frac{4}{5}}$

$\frac{1}{\frac{9}{5}} = \frac{5}{9}$

c) $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}$

Converge

$\frac{a_1}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{.5}$

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